Math 2X03 - Homework 1

Due: May 12, 2016

Chapters Covered on Homework - Chapter 15.1 - Chapter 15.4

1. (Chapter 15.2 # 20) Calculate the double integral

$$\iint_R \frac{x}{1+xy}\, dA \quad R = [0,1] \times [0,1]$$

2. (Chapter 15.2 # 21) Sketch the solid whose volume is given by the iterated integral

$$\int_0^1 \int_0^1 (4 - x - 2y)\, dy\, dx$$

3. Find the area of the region $D = \left\{(x,y) \mid 1 \le x \le 2,\ \ln x \le y \le x^2\right\}$

4. The **Average value** of a function f of two variables defined on a rectangle R is defined to be

$$f_{ave} = \frac{1}{A(R)} \iint_R f(x,y)\, dA$$

Find the average value of the function $f(x,y) = xy\sin(x^2 y)$ over the rectangle $R = [0,1] \times [0, \frac{\pi}{2}]$.

5. Let D denote the region R bounded by the curves $x^2 + y^2 = 16$ and $x^2 + y^2 = 25$ in the first quadrant. Evaluate the integral

$$\iint_R x + \sqrt{x^2 + y^2}\, dA$$

6. Evaluate

$$\iint_D y^3\, dA$$

where D is the triangular region with vertices $(0,2), (1,1), (3,2)$.

7. Evaluate the double integral

$$\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy\, dx$$

Hint: Change the order of integration (i.e sketch the region of integration and you will notice that the rgion is both Type I and Type II. Rewrite it as an integral over a Type II region.)

8. (Chapter 15.3 # 53) Evaluate the integral by reversing the order of integration

$$\int_0^1 \int_{\arcsin(y)}^{\pi/2} \cos x \sqrt{1 + \cos^2 x}\, dx\, dy$$

9. Evaluate the double integral

$$\iint_D e^{x/y}\, dA\,;\ D = \{(x, y)\,|\, 1 \le y \le 2,\ y \le x \le y^3\}$$

10. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$, above the xy-plane, and inside the cylinder $x^2 + y^2 = 2x$.

    Remark: You are expected to know that graphs of quadric surfaces. To review read through Chapter 12.6, and if you have any questions feel free to stop by during office hours and I am more than happy to help.

11. (Chapter 15.4 #29) Evaluate the iterated integral by converting to polar coordinates:

$$\int_{-3}^{3} \int_{0}^{\sqrt{9-x^2}} \sin(x^2 + y^2)\, dy\, dx$$

12. (Chapter 15.4 #40)

    (a) We define the improper integral (over the entire plane $\mathbb{R}^2$

$$\begin{aligned} I &= \iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\, dA = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, dy\, dx \\ &= \lim_{a\to\infty} \iint_{D_a} e^{-(x^2+y^2)}\, dA \end{aligned}$$

    where D_a is the disk with radius a and center the origin.

    Show that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, dA = \pi$$

    (b) An equivalent definition of the improper integral in part (a) is

$$\iint_{\mathbb{R}^2} e^{-(x^2+y^2)}\, dA = \lim_{a\to\infty} \iint_{S_a} e^{-(x^2+y^2)}\, dA$$

    where S_a is the square with vertices $(\pm a, \pm a)$. Use this to show that

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx \int_{-\infty}^{\infty} e^{-y^2}\, dx = \pi$$

    (c) Deduce that

$$\int_{-\infty}^{\infty} e^{-x^2}\, dx = \sqrt{\pi}$$

    (d) By making the change of variable $t = \sqrt{2}\,x$, show that

$$\int_{-\infty}^{\infty} e^{-x^2/2}\, dx = \sqrt{2\pi}$$

    (This is a fundamental result for probability and statistics).